IMC 2026

First Day, July 29, 2026

Problem 1. Show that the equation $\cos(\cos x) = \sin(\sin x)$ has no real solutions.

(10 points)

Problem 2. Let n be a positive integer. Suppose that A and B are $n \times n$ matrices with real entries such that

$$A^\top A + BB^\top = AB + BA,$$

where $X^\top$ denotes the transpose of matrix X.

Does this imply that $AB = BA$?

(10 points)

Problem 3. Consider a deck of $n \geq 2$ cards labeled $1, 2, \ldots, n$. An *alternating shuffle* of the deck is performed as follows. We split the deck into two non-empty stacks. We then sort the first stack in increasing order, and the second stack in decreasing order. Finally, we alternately take cards from the first and second stacks (starting with the first). If one of the stacks runs out, the remaining cards from the other stack are placed at the end. How many different final orders of the deck can be obtained in this way?

Example: Suppose $n = 6$, the first stack is $A = (1, 3)$, and the second stack is $B = (6, 5, 4, 2)$. Then the order resulting from the alternating shuffle is (1, 6, 3, 5, 4, 2).

(10 points)

Problem 4. Let $x_1 > 0$. Define the sequence $\{x_n\}$ by the recurrence

$$x_{n+1} = \arctan\left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right) \quad \text{for all } n \geq 1.$$

Find $\lim\limits_{n\to\infty} x_n\sqrt{\ln n}$, where $\ln x$ denotes the natural logarithm of x.

(10 points)

Problem 5. Prove that there exists a constant $C > 0$ such that for every pair A, B of positive integers, there is a real polynomial $p(x)$ with

$$p(0)^2 > \sum_{i=1}^{A} p(-i)^2 + \sum_{i=1}^{B} p(i)^2 \quad \text{and} \quad \deg p < C\sqrt{AB}.$$

(10 points)