

IMC 2020 Online

Day 2, July 27, 2020

Problem 5. Find all twice continuously differentiable functions $f : \mathbb{R} \rightarrow (0, +\infty)$ satisfying

$$f''(x)f(x) \geq 2(f'(x))^2$$

for all $x \in \mathbb{R}$.

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Problem 6. Find all prime numbers p for which there exists a unique $a \in \{1, 2, \dots, p\}$ such that $a^3 - 3a + 1$ is divisible by p .

Géza Kós, Loránd Eötvös University, Budapest

Problem 7. Let G be a group and $n \geq 2$ be an integer. Let H_1 and H_2 be two subgroups of G that satisfy

$$[G : H_1] = [G : H_2] = n \quad \text{and} \quad [G : (H_1 \cap H_2)] = n(n - 1).$$

Prove that H_1 and H_2 are conjugate in G .

(Here $[G : H]$ denotes the *index* of the subgroup H , i.e. the number of distinct left cosets xH of H in G . The subgroups H_1 and H_2 are *conjugate* if there exists an element $g \in G$ such that $g^{-1}H_1g = H_2$.)

Ilya Bogdanov and Alexander Matushkin, Moscow Institute of Physics and Technology

Problem 8. Compute

$$\lim_{n \rightarrow \infty} \frac{1}{\log \log n} \sum_{k=1}^n (-1)^k \binom{n}{k} \log k.$$

(Here \log denotes the natural logarithm.)

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