

IMC2026: Problems on Day 1

Problem 1. Show that the equation $\cos(\cos x) = \sin(\sin x)$ has no real solutions.

Alexander Slávik, Charles University, Prague

Problem 2. Let n be a positive integer. Suppose that A and B are $n \times n$ matrices with real entries such that

$$A^T A + B B^T = AB + BA,$$

where X^T denotes the transpose of matrix X .

Does this imply that $AB = BA$?

Nikolaos Kolliopoulos, University of Cyprus

Problem 3. Consider a deck of $n \geq 2$ cards labeled $1, 2, \dots, n$. An *alternating shuffle* of the deck is performed as follows. We split the deck into two non-empty stacks. We then sort the first stack in increasing order, and the second stack in decreasing order. Finally, we alternately take cards from the first and second stacks (starting with the first). If one of the stacks runs out, the remaining cards from the other stack are placed at the end. How many different final orders of the deck can be obtained in this way?

[0.5cm] *Example:* Suppose $n = 6$, the first stack is $A = (1, 3)$, and the second stack is $B = (6, 5, 4, 2)$. Then the order resulting from the alternating shuffle is $(1, 6, 3, 5, 4, 2)$.

Daniel Volostnov, Neapolis University Paphos, Cyprus

Problem 4. Let $x_1 > 0$. Define the sequence $\{x_n\}$ by the recurrence

$$x_{n+1} = \arctan\left(\frac{x_1 + x_2 + \dots + x_n}{n}\right) \text{ for all } n \geq 1.$$

Find $\lim_{n \rightarrow \infty} x_n \sqrt{\ln n}$, where $\ln x$ denotes the natural logarithm of x .

Wanlong Han, Henan, China

Problem 5. Prove that there exists a constant $C > 0$ such that for every pair A, B of positive integers, there is a real polynomial $p(x)$ with

$$p(0)^2 > \sum_{i=1}^A p(-i)^2 + \sum_{i=1}^B p(i)^2 \quad \text{and} \quad \deg p < C\sqrt{AB}.$$

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