IMC2015: Problems on Day 2

6. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,(n+1)} < 2.$$

Proposed by Ivan Krijan, University of

Sol

7. Compute

$$\lim_{A\to+\infty} \frac{1}{A} \int_1^A A^{\frac{1}{x}}\,dx\,.$$

Proposed by Jan Šustek, University of

Hint Sol

8. Consider all 26^{26} words of length 26 in the Latin alphabet. Define the *weight* of a word as $1/(k+1)$, where k is the number of letters not used in this word. Prove the sum of the weights of all words is 3^{75}.

Proposed by Fedor Petrov, St. Petersburg State Uni

Sol

9. An $n \times n$ complex matrix A is called \emph{t-normal} if $AA^t = A^tA$ where A^t is the transpose of A. For each n, determine the maximum dimension of a linear s of complex $n \times n$ matrices consisting of t-normal matrices.

Proposed by Shachar Carmeli, Weizmann Institute of S

Sol

10. Let n be a positive integer, and let $p(x)$ be a polynomial of degree n with integer coefficients. Prove that

$$\max_{0\le x\le 1} |p(x)| > \frac{1}{e^n}.$$

Proposed by Géza Kós, Eötvös University, Bu

Hint Sol

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