# IMC2015: Problems on Day 1

**1.** For any integer $n \geq 2$ and two $n \times n$ matrices with real entries $A,\ B$ that satisfy the equation

$$A^{-1} + B^{-1} = (A+B)^{-1}$$

prove that $\det(A) = \det(B)$.

Does the same conclusion follow for matrices with complex entries?

Proposed by Zbigniew Skoczylas, Wroclaw University of Tech

Hint Sol

**2.** For a positive integer $n$, let $f(n)$ be the number obtained by writing $n$ in binary and replacing every 0 with 1 and vice versa. For example, $n = 23$ is 10111 in bina $f(n)$ is 1000 in binary, therefore $f(23) = 8$. Prove that

$$\sum_{k=1}^{n} f(k) \leq \frac{n^2}{4}.$$

When does equality hold?

Proposed by Stephan Wagner, Stellenbosch Uni

Hint Sol

**3.** Let $F(0) = 0$, $F(1) = \frac{3}{2}$, and $F(n) = \frac{5}{2}F(n-1) - F(n-2)$ for $n \geq 2$.

Determine whether or not $\sum_{n=0}^{\infty} \frac{1}{F(2^n)}$ is a rational number.

Proposed by Gerhard Woeginger, Eindhoven University of Tech

Hint Sol

**4.** Determine whether or not there exist 15 integers $m_1, \ldots, m_{15}$ such that~

$$\sum_{k=1}^{15} m_k \cdot \arctan(k) = \arctan(16). \qquad (1)$$

Proposed by Gerhard Woeginger, Eindhoven University of Tech

Hint Sol

**5.** Let $n \geq 2$, let $A_1, A_2, \ldots, A_{n+1}$ be $n+1$ points in the $n$-dimensional Euclidean space, not lying on the same hyperplane, and let $B$ be a point strictly inside the co hull of $A_1, A_2, \ldots, A_{n+1}$. Prove that $\angle A_iBA_j > 90^\circ$ holds for at least $n$ pairs $(i,j)$ with $1 \leq i < j \leq n+1$.

Proposed by Géza Kós, Eötvös University, Bu

Hint Sol



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